About the Dependence of the Currency Exchange Rate at Time and National Dividend, Investments Size, Difference Between Total Demand and Supply

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The time dependence of the currency exchange rate K treated as a function of national dividend, investments and difference between total demand for a goods and supply is considered. To do this a proposed earlier general algorithm of economic processes describing on the basis of the equations for K like the equations of statistical physics of open systems is used. A number of differential equations (including nonlinear ones too) determining the time dependence of the exchange rate (including oscillations) is obtained.

1. L.Ya.Kobelev et al. [1] in order to describe economic phenomena offered a system of nonlinear differential equations based on the mathematical methods of statistical physics of open systems ([2]).(see also [3]- [11]). The present paper is devoted to consideration of a model example of calculation of time dependence of national currency exchange rate treated as a function of national dividend F, investments C and total difference between demand and supply (with all the values being measured in national currency units). The modality of the example considered is in neglecting of dependencies of national currency upon the others, not mentioned economic variables, so the main purpose of this paper is not so to elucidate real economic laws and regularities (although this also takes place) as to illustrate the possibilities of the offered in the named paper method if applied to a concrete problem.

2. Thus, to show the advantages of the method used consider the dependence of the currency exchange rate upon investments C, difference between total demand and supply P, and national dividend F. Let K be a variation of the currency exchange rate per time unit. In this case the equations describing the exchange rate as a function of C, P and F takes the form of the simplified equations (3), (4), (7) of the paper [1]

$$\frac{dK}{dt}=\frac{\partial K}{\partial t}+\frac{\partial K}{\partial C}\frac{dC}{dt}+\frac{\partial K}{\partial P}\frac{dP}{dt}+\frac{\partial K}{\partial F}\frac{dF}{dt}=I_C \tag{1}$$

$$\begin{aligned}I_C=\varphi(K)+\frac{\partial}{\partial C}(D_C\frac{\partial}{\partial C}K)+\\+\frac{\partial}{\partial p}(D_P\frac{\partial}{\partial P}K)+\frac{\partial}{\partial F}(D_F\frac{\partial}{\partial F}K)\end{aligned} \tag{2}$$

The equations describing time dependencies of C, P, F and the dependence of the currency exchange rate upon them write down as follows:

$$\frac{dK}{dx_i}=F_i(x_i,K)+\frac{\partial}{\partial x_j}[D_{j\beta}\frac{\partial}{\partial x_\beta}(x_i-A_jx_i)] \tag{3}$$

$$\frac{dx_i}{dt}=\varphi(x_j,\dot{x_j},K,\dot{K},\frac{\partial K}{\partial x_j}) \tag{4}$$

where $i=1,2,3$ ($x_1=C$, $x_2=P$, $x_3=F$), F_i and ϕ are nonlinear functions of their own arguments. In eqs. (2)-(3) the terms containing D_C, D_P and D_F coefficients describe "diffusion" in the space of the exchange rate alteration variables C, P, F, i.e., the distribution of K over these variables, and the $\phi(K)$ terms describe the dependence of $\frac{dK}{dt}$ upon the processes regulating the velocity of the exchange rate alteration (in particular, a currency exchange rate relaxation, bistable states existence etc.).

3. In the simplest case chose $_C=0$, that is, consider the equation

$$\frac{dK}{dt}=\frac{\partial K}{\partial t}+\frac{\partial K}{\partial C}\frac{dC}{dt}+\frac{\partial K}{\partial P}\frac{dP}{dt}+\frac{\partial K}{\partial F}\frac{dF}{dt}=0 \tag{5}$$

The equality I_C=0 corresponds to neglecting by the currency exchange rate alteration smoothing processes influence on $\frac{dK}{dt}$. The $\frac{\partial K}{\partial C}$, $\frac{\partial K}{\partial P}$ and $\frac{\partial K}{\partial F}$ derivatives in (5) are determined, in difference with kinetic theory in statistical physics, by eqs.(3) and the derivatives $\dot{x}_i$ are governed by (4).

4. Consider now, for the sake of simplicity, the case when the influence of P and F on the currency exchange rate alteration is very small and may not be taken into account. Then

$$\frac{\partial K}{\partial t}+\frac{\partial K}{\partial C}\frac{dC}{dt}=0 \tag{6}$$

To define $\frac{\partial K}{\partial C}\frac{dC}{dt}$ we use a reduced equations (1)-(4)

$$\frac{dC}{dt}=a\int_0^t K(t)dt \tag{7}$$

$$\frac{\partial K}{\partial C}=\alpha \tag{8}$$

where α in (7) is a velocity of changing in investments attributed to the sum of the currency exchange rate alteration over the time interval of t and α in (8) is the changing in the currency exchange rate alteration per investments unit.

Assume then that these values do not vary significantly, that is $\alpha = const$ and $\frac{dC}{dt} = b = const$. In this case we will have $\frac{\partial K}{\partial t} = K_1 = ab$ and $K = K_1 t + K_2$ ($K_2 = const.$). So K will increase or decrease, depending upon the sign of K_1. In case of (7), differentiating (6) in time and using (13) from paper [1] and (8) yield

$$\frac{\partial^2 K}{\partial t^2} + \omega_0^2 K = 0$$

where $\omega_0 = \sqrt{a\alpha}$. The solution of (9) has the form

$$K = K_0 \sin \omega_0 t$$

where K_0 is constant. So with the assumption made, the currency exchange rate alteration K varies periodically in time with the frequency of $\sqrt{a\alpha}$. If the dependence of K upon the investments made is weak, the frequency of oscillations will be small and K will take the form ($\omega_0 t \ll 1$)

$$K = K_0 \sqrt{a\alpha} t + ... \qquad \sqrt{a\alpha} t \ll 1$$

At times large enough, nevertheless, the periodicity of the currency exchange rate varying with time will be observed.

5. Introduce now into the right-hand side of eq.(5) a parameter characterizing the deviation of K from an equilibrium: $I_c = \frac{K - K_0}{\tau(K)}$, where $\tau(K)$ is the relaxation time for the currency exchange rate alteration if diverged from a stationary state K_0 (in general case τ is a function of K, $\dot{K}$, C, F etc.)
Then instead of (6) we have

$$\frac{\partial K}{\partial t} + \frac{\partial K}{\partial C}\frac{dC}{dt} = \frac{K - K_0}{\tau(K)} \qquad (9)$$

Assuming $K_0 = const$ and $\tau = const$ differentiating in time, if eqs. (7) and (8) are valid, yields:

$$\ddot{K} + \omega_0^2 K - \frac{1}{\tau}\dot{K} = 0$$

whose solution is

$$K = k_0 e^{-\frac{t}{\tau}} \cos \omega_0 t$$

In this case the oscillations of the currency exchange rate changing will damp tending to zero when $t \ll \tau$.

6. Take into account in equation (9) the difference P between total demand and supply influence on the exchange rate alteration, determining $\frac{dP}{dt}$ and $\frac{\partial K}{\partial P}$ from the equations (a special case of (1) and (4))

$$a)\ \frac{dP}{dt} = \gamma \dot{K}, \quad \frac{\partial K}{\partial P} = \beta K^2$$

$$b)\ \frac{dP}{dt} = \beta K^2, \quad \frac{\partial K}{\partial P} = \gamma \dot{K}$$

with linear (a) or nonlinear (b) dependence of $\frac{dP}{dt}$ upon $\dot{K}$ or K correspondingly. Then instead of (9) we get

$$\frac{\partial K}{\partial t} + a\alpha \int_0^t K(t')dt' + \beta\gamma K^2 \dot{K} = \frac{K - K_0}{\tau(K)} \qquad (10)$$

or, after differentiating with respect to time

$$(1 + \beta\gamma K^2)\ddot{K} - (\frac{1}{\tau} - 2\beta\gamma K\dot{K})\dot{K} + \omega_0^2 K = 0 \qquad (11)$$

If $\beta\gamma K^2 \ll 1$ and $2\beta\gamma K\dot{K} \gg \frac{1}{\tau}$, then (10) reduces to

$$\ddot{K} + 2\delta(K\dot{K})\dot{K} + \omega_0^2 K = 0 \qquad (12)$$

where

$$\delta(K\dot{K}) = \beta\gamma K\dot{K}$$

Eq. (11) is an equation of oscillations with a nonlinear friction $2\delta(K\dot{K})$ and its solutions contains all the peculiarities of nonlinear systems. Take into consideration in (10) the term $\frac{\partial}{\partial P}(D_P \frac{\partial}{\partial P})K$, assuming $D_P = const$. This yields a change in coefficient at in (12) in the case (a)

$$(1 + \beta\gamma K^2)\ddot{K} - (\frac{1}{\tau} - 2\beta\gamma K\dot{K} + D_P \beta^2 K^2)\dot{K} + \\ + \omega_0^2 K = 0 \qquad (13)$$

and appearance of a term containing $\frac{d^3K}{dt^3}$ in the case (b)

$$\gamma^2 D_P^2 \frac{d^3 K}{dt^3} - (1 + \beta\gamma K^2)\ddot{K} - \\ -2\beta(K\dot{K})\dot{K} - \omega_0^2 K = 0$$

7. One may take into account in (10) the terms containing $\frac{\partial K}{\partial F}\frac{dF}{dt}$ assuming, for example, $D_F = const$, and (a special case of (1)-(4))

$$a)\ \frac{\partial K}{\partial F} = b\int_0^t K(t')dt', \quad \frac{dF}{dt} = d = const \qquad (14)$$

$$b)\ \frac{\partial K}{\partial F} = d = const, \quad \frac{dF}{dt} = \int_0^t K(t')dt'$$

Eqs. (13) then takes the from ((14a) case)

$$(1 + \beta\gamma K^2)\ddot{K} - (\frac{1}{\tau} - 2\beta\gamma K\dot{K} + D_P \beta^2 K^2)\dot{K} + \\ [(\omega_0^2 + bd)K - D_F \beta^2 \int_0^t K(t')dt'] = 0 \qquad (15)$$

and after differentiating in t

$$(1 + \beta\gamma K^2)\frac{d^3K}{dt^3} - (\frac{1}{\tau} - 2\beta\gamma K\dot{K} + D_P \beta^2 K^2)'\dot{K} + \\ +(\frac{1}{\tau} - 2\beta\gamma K\dot{K} + D_P \beta^2 K^2)\ddot{K} + \qquad (16) \\ +[(\omega_0^2 + bd)\dot{K} - D_F \beta^2 K(t)] + 2\beta\gamma K\dot{K}\ddot{K} = 0$$

Using (14b) one obtains

$$(1+\beta\gamma K^2)\ddot{K} - (\frac{1}{\tau}2\beta\gamma K\dot{K} + D_P\beta^2K^2)\dot{K} +$$
$$+(\omega_0^2+bd)K = 0$$

Consider finally a case of nonlinear dependence of I_C upon K. Let, remaining the assumptions of the previous paragraphs concerning the dependencies of K upon D, C and F, I_C has the form

$$I_C = \int_0^t (l_0 - lK^2)Kdt + \frac{K-K_0}{\tau} + D_P\frac{\partial^2 K}{\partial P^2} + \qquad (17)$$
$$+D_C\frac{\partial^2 K}{\partial C^2} + D_F\frac{\partial^2 K}{\partial F^2}$$

The choice of (17) corresponds to nonlinear type of $K(t)$ relaxation. Substituting I_C from (17) (e.g., for (14b) case) into (2) yields (using the proper equations for derivatives of K, C, P, F):

$$(1+\beta\gamma K^2)\ddot{K} + (2\beta\gamma K\dot{K} - D_P\beta^2K^2 - \frac{1}{\tau})\dot{K} + \qquad (18)$$
$$+[(\omega_0^2+bd) - (l_0 - lK^2)]K = 0$$

Eq. (18) has a bifurcation point at

$$K = \pm\frac{1}{\sqrt{l}}\sqrt{l_0 - \omega_0^2 - bd}$$

(which corresponds to the appearance of a bistable state for the currency exchange rate alteration) and a number of other interesting peculiarities as well (in particular, at $K = \pm\sqrt{\frac{1}{|\gamma\beta|}}$, $(\gamma\beta < 0)$,

$$atK = \frac{1}{D_P\beta^2}(-\beta\gamma\dot{K} \pm \sqrt{(\beta\gamma\dot{K})^2 + \frac{2D_P\beta^2}{\tau}},$$

$$at\dot{K} = \frac{D_P\beta^2K \pm \sqrt{D_P\beta^2K^2 - 8\beta\gamma(\omega_0^2 + bd - l_0 - lK^2)}}{4\gamma\beta},$$

etc.)

8. Note, that wide opportunities to chose the equations for $\frac{\partial K}{\partial C}$, $\frac{\partial K}{\partial P}$, $\frac{\partial K}{\partial F}$ and $\frac{dC}{dt}$, $\frac{dP}{dt}$, $\frac{dF}{dt}$ and I_C are not exhausted by the selection used in the example considered and the concrete forms of the equations are determined by the state of the economic processes and correlations between them. For example, one may treat as x_i variables in the case considered realized Y_e and produced Y_l gross national products, the total sum of money in use, the number of workable population N, absolute level of unemployment δN, the part of gross national product used by the state (these variables were used by Bystrai ([3]) while defining the economic entropy.

CONCLUSION

A general algorithm of describing economic processes basing on the equations of statistical physics of open systems developed by Kobelev L.Ya. et al. [1] was used to describe the time dependence of the national currency exchange rate as a function of national dividend, investments size and difference between total demand for the goods and its supply. A number of nonlinear differential equations describing the time dependence of exchange rate (in particular, oscillations in different cases) were obtained.

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